**Louise Spear-Swerling**

**Developing Automatic Recall of Addition and Subtraction Facts**

**June 2006**

Automatic recall of basic math facts, sometimes termed math fluency, is generally considered to be a key foundation for higher-level math skills. Basic math facts are the answers to simple equations in addition (e.g., 3 + 2), subtraction (e.g., 9 – 5), multiplication (e.g., 6 x 4), and division (e.g., 15 ÷ 3). When children have automatic recall of facts, they can quickly retrieve answers from memory without having to rely on counting procedures, such as counting on fingers. Lack of automatic recall is a problem as children advance into the middle and later elementary grades, because the need to rely on laborious counting procedures creates a drain on mental resources needed for learning more advanced mathematics. Here the focus will be on automatic recall of addition and subtraction facts, which develops in normally-achieving youngsters by about the end of second grade.

Poor automatic recall of facts is very common among struggling students. One factor behind these difficulties may be that contemporary methods of math instruction, strongly influenced by the standards of the National Council of Teachers of Mathematics (NCTM), have tended to de-emphasize memorization of basic facts in favor of a greater emphasis on problem-solving and concept development. Adequate attention to promoting memorization of facts in the general education math curriculum may prevent or ameliorate problems with automatic recall in many students. However, some youngsters with learning disabilities may have considerable difficulty with automatic recall despite basic fact instruction that is adequate for developing memorization of facts in most children.

**What are the addition and subtraction facts?**

Addition facts involve single-digit addends with sums to 18 (9 + 9). Thus, 2 + 7, 9 + 6, and 8 + 3 are all addition facts, whereas 11 + 12 is not a fact because the addends are not single digits and the sum is greater than 18. Subtraction facts are the inverses of the addition facts. For example, 10 – 7 = 3, 16 – 8 = 8, and 5 – 0 = 5 are all subtraction facts, because they are all inverses of addition facts (7 + 3, 8 + 8, 5 + 0). However, 19 – 3 = 16 is not a subtraction fact, because 16 + 3 = 19 is not an addition fact. It is important for teachers to know which equations are facts, because practice activities on these particular equations are essential for developing automatic recall; for other (non-fact) equations, computational algorithms should be taught. For instance, rather than memorizing the answer to 11 + 12, children should be taught the algorithm for 2-digit addition without regrouping (i.e., align the numbers vertically in columns, add the ones column, then add the tens column).

**Important prerequisites for automatic recall**

Before attempting to develop automatic recall in children, teachers should ensure that the children have certain prerequisite skills and knowledge. Prerequisites for automatic recall of addition facts include conceptual understanding of addition, the ability to solve addition facts accurately under un-timed conditions, and understanding that the order of the addends does not change the sum (i.e., the commutative principle). Prerequisites for automatic recall of subtraction facts include conceptual understanding of subtraction and the ability to solve subtraction facts accurately under un-timed conditions. Children who lack any of these prerequisites should be taught the prerequisite skill first before the teacher attempts activities to promote memorization of facts. Also, it should be noted that children can be at substantially different levels for addition and subtraction, with the former usually outstripping the latter. For example, a given youngster might be ready to work on automatic recall of addition facts but still be struggling with the basic concept of subtraction.

**Activities to promote automatic recall**

Practice activities are very important in developing automatic recall. Many children will not spontaneously memorize facts solely from experience with manipulatives or problem-solving activities. Games that incorporate the use of facts---such as card games, board games, or computer games---are all helpful and generally palatable ways for children to gain additional practice with facts.

Fluency-based measures are useful in developing automatic recall as well as in monitoring the progress of individual children. With these measures, children are given a set of 30-50 written facts that they must solve within a particular time interval, usually one to three minutes. The teacher should begin with a subset of relatively easy facts, such as sums to 5 in addition. (To have 30 items on a page, facts may need to be randomly repeated.) When children have reached the speed goal on those facts, more difficult ones are added incrementally (e.g., sums to 7, then sums to 10, then sums to 14, and finally sums to 18). The usual speed goal for automatic recall of facts is 30 correct per minute. However, the “30 per minute” figure is based on oral responses; a written test imposes motor demands that may slow the responses of younger children or those with motor difficulties. If writing speed is an issue, then fact knowledge can be assessed orally, through timed flash card activities or having individual children give oral responses to fluency measures.

**Teaching number strategies**

Teaching number strategies can be a good interim step while children are working on developing automatic recall. Many struggling math students use burdensome “counting all” strategies for solving facts, which involve counting all numbers in the equation starting from 1. For example, a child who “counts all” in addition might solve 5 + 6 by making 5 lines on a piece of paper, then 6 more lines, then counting all of the lines together, starting from 1, to arrive at the answer 11. A child who “counts all” in subtraction might solve 14 – 9 by making 14 lines on paper, crossing off 9 lines, and then counting all of the remaining lines to obtain the answer 5. Though accurate, this approach is very time-consuming, especially for the larger facts such as sums in addition from 11 – 18. Number strategies involve more efficient counting procedures that enable children to count more quickly and to minimize the chance of errors.

A useful strategy for addition facts is counting up. With this strategy, children solve a fact by counting up from the larger number, not necessarily the first number, in the equation; for 5 + 6, children would start counting up from 6, 5 additional counts (7, 8, 9, 10, 11), to obtain the answer 11. A common error in counting up is for a child erroneously to count the first number in the sequence (in this case, 6), so that his or her answers are consistently off by 1. Teaching children to use a motor cue can be helpful in preventing this problem. For instance, the teacher can tell children to “put the larger number in their heads first” before beginning to count up; for 5 + 6, children would say “six” while touching their heads, then count up by 5 using their fingers. A number line can also be very effective in illustrating the “counting up” strategy.

In subtraction, children can use either counting up or counting down as strategies. When counting up in subtraction, children count up from the smaller number to the bigger number; the answer is the number of counts. For example, for 12 – 7, children count up from 7, raising a finger for each count (8, 9, 10, 11, 12), yielding an answer of 5. Counting down requires counting backwards from the larger number the number of counts indicated by the smaller number. For example, for 12 – 7, the child would count backwards from 12, seven counts (11, 10, 9, 8, 7, 6, 5); the answer is the final number in the sequence (in this case, 5). Again, a number line can be especially helpful in illustrating the “counting down” strategy.

For both addition and subtraction, being able to use a known fact to figure out an unknown fact is another useful strategy. For instance, if a child knows that 5 + 5 = 10, then 5 + 6 must be one more than 10, or 11; if a child knows that 10 – 1 = 9, then 10 – 2 must be one less than 9, or 8.

Explicit teaching of number strategies can be very beneficial, but a few caveats are in order. Effective use of strategies requires conceptual understanding of relationships among numbers; thus, as with automatic recall, teaching number strategies is premature if children are still struggling with basic concepts of addition or subtraction. Children should not be introduced to more than one new strategy at a time, and they should master and use a particular strategy consistently before being introduced to any additional strategies. It often takes time for children to acquire a number strategy and especially to employ a new strategy consistently in everyday work. If too many strategies are introduced too quickly, children can become very confused. Also, although number strategies are less arduous than “counting all,” they are not a substitute for automatic recall, because they still require some mental effort.

**Conclusion**

Automatic recall of addition and subtraction facts is an important basic skill, one that is frequently weak in youngsters with learning disabilities. Adequate attention to fact memorization in the general education curriculum may help to prevent difficulties with automatic recall in some students, whereas others may need more intensive teaching and additional opportunities for practice to develop automatic recall. Nevertheless, fact memorization should not be overemphasized in the math programs of students with learning disabilities. In particular, children should not be held behind in other areas of math while awaiting the development of automatic recall. For example, children with poor memorization of facts can be allowed to use a fact chart or a calculator for math activities whose primary focus involves problem-solving or concept development, while still continuing to work on automatic recall in other parts of the lesson. Development of automatic recall should be only one strand, although an important one, in a broader program of mathematics.

**Examples of sources**

Peer-reviewed journal articles:

* Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. The Elementary School Journal, 103, 51-73.
* Carnine, D. (1997). Instructional design in mathematics for students with learning disabilities. Journal of Learning Disabilities, 30, 130-141.
* Cawley, J., Parmar, R., Foley, T., Salmon, S., & Roy, S. (2001). Arithmetic performance of students: Implications for standards and programming. Exceptional Children, 67, 311-330.
* Cawley, J., Parmar, R., Shephard, T., & Smith, M. (1997). Item complexity and computation performance of students with disabilities. Learning Disabilities: A Multidisciplinary Journal, 8, 97-107.
* Christensen, C. A., & Cooper, T. J. (1991). The effectiveness of instruction in cognitive strategies in developing proficiency in single-digit addition. Cognition and Instruction, 8, 363-371.

Other helpful sources:

* Miller, S. P., & Mercer, C. D. (1998). Educational aspects of mathematics disabilities. In D. P. Rivera (Ed.), Mathematics education for students with learning disabilities: Theory to practice (pp. 81-96). Austin, TX: Pro-Ed.
* Rivera, D. P. (1998). Mathematics education and students with learning disabilities. In D. P. Rivera (Ed.), Mathematics education for students with learning disabilities: Theory to practice (pp. 1-32). Austin, TX: Pro-Ed.
* Shinn, M. R., & Bamonto, S. (1998). Advanced applications of curriculum-based measurement: “Big ideas” and avoiding confusion. In M. R. Shinn (Ed.), Advanced applications of curriculum-based measurement (pp. 1-31). New York: Guilford.
* Stein, M., Silbert, J., & Carnine, D. (2006). Designing effective mathematics instruction: A direct instruction approach (4th edition). Upper Saddle River, NJ: Merrill.

**LD Online links**

* [Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities](http://www.ldonline.org/article/5838)
* [Technology-Supported Math Instruction for Students with Disabilities](http://www.ldonline.org/article/6291)
* [Components of Effective Mathematics Instruction](http://www.ldonline.org/article/5588)

**Other helpful links**

* [Trends in Math Achievement: The Importance of Basic Skills](http://www.brookings.edu/views/speeches/loveless/20030206.htm)
* [Finding Common Ground in the U.S. Math Wars](http://www.science.tamu.edu/story3.asp?storyID=514)
* [Math Fluency](http://www.illinoisloop.org/md_math_fluency.html)

<http://ldonline.org>